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Cardinal Numbers and Confidence

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Queen Elsa's Bridge confidence

Confidence that numbers and mathematics can be trusted to work reliably is different from the confidence in one's own abilities that comes from success in learning. I like to think of the type of confidence that reassures a learner that the mathematics itself can be trusted as "Queen Elsa's Bridge confidence".

In a famous scene from the Disney movie *Frozen*, Queen Elsa creates a magical bridge and confidently runs across it (Frozen, 2013). If you are a parent of a child who enjoys Disney movies, you are probably already familiar with this scene. Otherwise, here is how you can find it: Go to Youtube, search for Frozen Song Let It Go, and start at about one and a half minutes into the video.

When learners don't have confidence that cardinal numbers can be trusted to work the same way every time, that's like seeing Queen Elsa's bridge and saying "There's no way I'm walking across that crazy thing."

These learners don't lack confidence in their own ability to walk without stumbling. What they lack is the confidence that Queen Elsa's bridge is solid enough to hold them up if they trust it enough to use it. When mathematician Francis Su says that we seek permanence because it is "a foothold on which we can rely" (2020, p. 95-98), he is talking about this kind of trust.

Of course, learners do need confidence in their own mathematical abilities. But the critical point here is that they also need confidence that they can trust in the mathematics itself.

Cardinal numbers work the same way on any set, no matter what kind of members the set has

This kind of confidence is key to being comfortable with the cardinal numbers that we use in arithmetic. A familiar definition of cardinal numbers states that "Cardinal numbers say how many of something there are, such as one, two, three, four, five" (Math Is Fun, 2018). Mathematicians use a different definition of cardinal numbers, which incorporates the familiar definition but also expands on it. To mathematicians, cardinality means that a "five" answering the question "How many cookies are on the plate?" and a "five" answering the question "How many pencils are on the table?" represent the same amount, because *the cardinal number five is precisely that which is common to the set of cookies, the set of pencils, and to all other sets having that many objects*. The idea here is to construct a "a notion of the relative size or 'bigness' of a set without reference to the kind of members which it has" (quotation from "Cardinal Number", Wikipedia, 2020).

What do children need to know about cardinal numbers?

It would not make sense to try to teach children the "mathematician meaning" of cardinal numbers by using a verbal explanation such as the one about cookies and pencils, which would be too abstract for young children to follow. But children will be helped tremendously if they can develop an intuition for the practical application, which is that cardinal numbers work the same way on any set.

For example, one thing the "mathematician meaning" tells us is that because 2 and 3 are cardinal numbers, "2 + 3" will always be the same amount, no matter what the two things and the three things are, or how they are arranged. "2 + 3 = 5" is true for two and three of anything, and we can feel confident that it always will be true.

Children who know that counting shows how many can still lack confidence about using cardinal numbers

In developmental psychology, children are considered "cardinal principle knowers" if their responses in certain tasks indicate that they know that the last number word used to count a set of five or more objects tells how many objects are in the set (Wynn, 1990; Wynn, 1992; Sarnecka and Carey, 2008; Davidson et al., 2012). Researchers (and teachers!) are aware that "cardinal principle knowers" may not understand all the problems that can be solved by counting, or why counting works to solve these problems (Fuson, 1991; Richardson, 2002, p. 27; Sarnecka and Carey, 2008; Davidson et al., 2012). This topic is the focus of current developmental psychology research.

The related point I want to make here is that children who know the cardinal number of objects in a set they have just counted may not feel confident generalizing that understanding to other sets. To see what I mean, let's revisit the "crayons and blocks" scenario. Cardinal principle knowers know that when they counted the crayons, there were five. From my experience helping learners, my sense is that some cardinal principle knowers do not yet have an intuition for the "mathematician meaning" of cardinal numbers. What this means is that these learners do not feel confident that the "two" and "three" that worked one way for the crayons can be trusted to work the same way the next time. Without that confidence, when it is time to think about the blocks they need to count one by one all over again.

Developmental psychologist Ann Dowker has called a closely related phenomenon the ability to use the "Identity principle": *If an arithmetical operation produces a given result, then the repetition of the same arithmetical principle will produce the same result* (Dowker, 2009). Dowker has used a test of the Identity principle (e.g., if a learner is told that 8 + 6 = 14, then that learner can automatically give the answer "14", without calculating, if asked "What is 8 + 6?") in research studies showing that some children at early stages of arithmetic learning do not appear to take advantage of this principle to help them solve problems (Dowker, 2009; Dowker, 2014).

Dowker's test is different from the crayons and blocks scenario in that it uses an abstract equation instead of real-world objects. Fundamentally, however, the crayons and blocks scenario is a way to assess Dowker’s Identity principle in children who can count to add but do not yet know the fact 2 + 3 = 5.

Dowker has commented that it is surprising that the Identity principle has been so rarely studied in children, since it is so foundational in arithmetic (Dowker, 2014). Similarly, it seems surprising that more attention has not been given to the relationship between Dowker’s Identity principle (or closely related scenarios) and mathematical confidence.

To become a high achiever in arithmetic, children need to develop an intuition for the “mathematician meaning” of cardinal numbers. This obviously does not mean that they need to understand complex concepts about cardinality, which can be tricky even for college mathematics majors. It just means that they need to gain confidence, so they can be sure the numbers will work the same way every time.

Your words can help children gain confidence using cardinal numbers

When learners start trusting numbers to be reliable, they are on their way to success in mathematics. You can help learners acquire this kind of trust with simple language like this: “We just counted these two crayons and these three crayons. And we know that all together, there were five crayons. What about these two red blocks and these three blue blocks? Do we have to count them to know how many there are all together, or can we just use the “five” answer from the crayons?” Of course, the right answer is that they can use the answer from the crayons.

Another way I like to talk about this concept is to wait until a child has just counted out any addition fact that was new to him — for example, counting out six and four and discovering that it is ten — and then ask, “Is 6 plus 4 only sometimes 10, or always 10?” Seeing the child’s face change as he first contemplates the idea of *always* is a guaranteed “make a teacher’s day” moment.

A child who asks you questions like "If 3 plus 5 is 8, does that mean 8 minus 5 is 3?" may be wondering whether she can trust the results of her own mathematical reasoning. This child is giving you an opportunity to boost her confidence. You can answer, "Yes, it definitely means that. You can be absolutely sure, if 3 plus 5 is 8, 8 minus 5 will always be 3." Answers like "figure it out for yourself" may be appropriate at other times, but right now, a child is asking you to reassure her that the creative step she took was safe. You should give her the reassurance she needs.

Games and activities can help, too

These games and activities [that is, the freely downloadable resources that the author has created, available at ReckonMath.com where this working paper appears] can also help learners start developing confidence in cardinal numbers. The counting activity "For finding out how many, the order doesn't matter" teaches explicitly that the cardinal number telling how many are in a set does not change when a different counting order is used. The "Which group doesn't belong?" counting activities teach explicitly that a cardinal number applies the same way to any set of that size, no matter what kinds of objects are in the set. The "Many ways to show [number] and [number]" games in Number Properties teach explicitly that a cardinal number applies the same way to any set of that size, no matter how the objects in the set are arranged. And throughout this program, the repeated experiences of success using the cardinal numbers in different games helps learners get in the habit of trusting that the cardinal numbers work the same way every time.

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